Robust Bayesian Estimators For Survival Function Under Prior Data Conflict With Practical Application in the Health Side

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Abstract

The analysis of survival functions is concerned with knowing how long humans will survive, This means studying and analyzing the time from the beginning of the disease to the end point of death. For example, the survival function of patients with heart attacks was studied and analyzed at Al Manathira General Hospital. The Weibull distribution was used to match the real data. The scale parameter & survival function have been estimated for Weibull distribution with have two parameters, this distribution was used in two cases, prior data unconflict & prior data conflict. A regular Bayes method & robust Bayes were used for estimation. We used inverse gamma distribution as a prior where it is a conjugate prior for Weibull distribution. Two simulation experiments have been used; the first experiment used was prior data unconflict where the regular Bayes method is the best for estimating the scale & survival function by using the integrated mean square error (IMSE) as a criterion for comparing. The second experiment is in the case of prior data conflict. The results showed that the robust Bayes method is the best for estimation of the scale parameter & survival function by using (IMSE).

Keyword: *Robust Bayesian, Prior data conflict, Survival function, iLuck Model, Regular Bayesian*

Introduction

The statistical inference in the Bayesian method also relies on the prior information or the so-called prior distribution so that the prior distribution is combined with the distribution of observations according to the base of the Bayes rule so we get the posterior distribution from here, a problem might appear, which is the prior data conflict (prior data are the default values For the prior distribution parameters), [6]. In the sense that the prior data does not necessarily correspond with the views or sample under study, and because of that you should know the existence of this problem or not when using the methods of Bayesian estimation, & to know the existence of this problem by modeling the parameters of the prior distribution, [8]. So that the prior distribution should be the conjugate prior, & then after modeling the prior distribution parameters we produce the standard deviation of the prior distribution & the standard deviation of the posterior distribution, [11],. If the value of the standard deviation of the prior distribution is greater than the standard deviation of the posterior distribution, then there is a problem, [10], Hence, the main objective of our research is to obtain the best estimate of the survival function under prior data conflict by addressing this problem by assuming a set of prior information to obtain a set of prior distributions & thus we will obtain a set of posterior distributions, [7,5]After that, we obtain the estimates that are more efficient & accurate so that the method is called the robust Bayesian estimation, where a two-parameter Weibull distribution will be used to estimate the parameter of scale parameter & survival function because it is considered to be the most common distributions of survival models, [9] So that the use of the usual Bayesian method & the robust Bayesian method for estimating the scale parameter & survival function & the IMSE will be used to compare these methods.

The estimation of methods

We will estimate the survival function for Weibull distribution & the scale parameter λ & consider the shape parameter β is known, in the Bayesian & robust Bayesian method as shown below:

$$f(t) = \frac{\beta}{\lambda} t^{\beta-1} e^{-\frac{t^\beta}{\lambda}} \qquad (1)$$

The prior distribution of the parameter λ will be used, which is inverse gamma:

$$f(\lambda \backslash a, b) = \frac{b^a}{\Gamma(a)} \lambda^{-a-1} e^{-\frac{b}{\lambda}} \qquad (2)$$

Then we get the posterior distribution as follows:

$$f(\lambda/t) = \frac{\left(\sum t_i^\beta + b\right)^{(a+n)}}{\Gamma(a+n)} \lambda^{-(a+n)-1} e^{-\frac{(\sum t_i^\beta + b)}{\lambda}} \qquad (3)$$

The above equation is the posterior distribution of parameter λ, & according to the quadratic loss function mean that the posterior distribution represents the Bayes estimator of parameter λ as shown below [3].

$$\hat{\lambda} = \frac{(b + \tau(t))}{a+n} \qquad (4)$$

$$where\ \tau(t) = \sum t^\beta$$

2-1-2: Bayesian Estimation for Survival function

The estimation of the survival function & it is based on the quadratic loss function as shown below by [2]:

$$\hat{S}(t) = \int_0^\infty S(t)\, h(\lambda/t)\, d\lambda$$

$$\hat{S}(t) = \left(\frac{b+\tau(t)}{b+\tau(t)+t^\beta}\right)^{a+n} \qquad (5)$$

Robust Bayesian Method

Checking for prior data conflict

Suppose that we have a sample distributed with the Weibull distribution & concisely t ~ wei (β, λ) as shown below [11, 12]:

$$f(t \backslash \lambda, \beta) = \frac{\beta}{\lambda} t^{\beta-1} e^{-\frac{t^\beta}{\lambda}}$$

The prior distribution of the scale parameter λ is inverse gamma because it is conjugate prior as shown below:

$$f(\lambda \backslash a, b) = \frac{b^a}{\Gamma(a)} \lambda^{-a-1} e^{-\frac{b}{\lambda}}$$

Then the prior parameters need to update so that it is $(n^0 > 1, y^0 > 0)$ instead of the parameters (a, b), through two methods, we get the prior distribution as shown below:

$$E(\lambda/a, b) = y^0 = \frac{b}{a-1} = \frac{b}{n^0} \Rightarrow b = n^0 y^0\ , n^0 = a - 1 \Rightarrow a = n^0 + 1$$

After that, the prior distribution is obtained by the parameters n^0, y^0, through testing the problem of prior-data conflict as shown below:

$$f(\lambda/n^0y^0) = \frac{(n^0y^0)^{n^0+1}}{\Gamma(n^0+1)} \lambda^{-(n^0+1)-1} e^{-\frac{n^0y^0}{\lambda}} \quad (6)$$

y^0: The prior guessing for the scale parameter λ.

n^0: The prior guessing to sample size n.

The equation above represents the distribution of prior by parameters n^0, y^0, which is also inverse gamma. After we obtain the prior distribution with parameters n^0, y^0, we derive the standard deviation of this distribution.

$$M_r = \frac{(n^0y^0)^r}{\Gamma(n^0+1)} \Gamma(n^0 - r + 1) \quad (7)$$

$$s.d\ prior = \sqrt{\frac{(y^0)^2}{1-\frac{1}{n^0}}} \quad (8)$$

The equation above represents the standard deviation of the prior distribution, & then the posterior distribution is derived as shown below:

$$f(\lambda\backslash t) = \frac{(n^0y^0+\tau(t))^{n^0+n+1}}{\Gamma(n^0+n+1)} \lambda^{-(n^0+n+1)-1} e^{-\frac{(n^0y^0+\tau(t))}{\lambda}} \quad (9)$$

After the posterior distribution is obtained & according to the equation above we derive the standard deviation for the posterior distribution as shown below:

$$M_r = \frac{(n^0y^0+\tau(t))^r}{\Gamma(n^0+n+1)} \Gamma(n^0 + n - r + 1) \quad (10)$$

$$s.d\ posterior = \sqrt{\frac{(n^0y^0+\tau(t))^2}{(n^0+n)(n^0+n-1)}} \quad (11)$$

Equation (13) represents the standard deviation of the posterior distribution to compare the standard deviation of the prior distribution of parameters n^0, y^0 with the standard deviation of the posterior distribution. If the standard deviation of the prior distribution is greater than the standard deviation of the posterior distribution, this means a prior-data conflict. This problem is needed to solve through the following steps.

A second way to obtain variance for posterior distribution is through the steps below:

As $f(\lambda/t, n^0, y^0) = f(\lambda/n^n y^n)$ It means :

$$f(\lambda/t, n^0, y^0) = f(\lambda/n^n y^n) = \frac{(n^n y^n)^{n^n+1}}{\Gamma(n^n+1)} \lambda^{-(n^n+1)-1} e^{-\frac{n^n y^n}{\lambda}} \quad (12)$$

The variance of this distribution is:

$$V(\lambda/n^n, y^n) = \frac{(y^n)^2}{1-\frac{1}{n^n}}$$

Therefore, the standard deviation of the prior distribution & the standard deviation of the posterior distribution can be written according to the following formula:

$$s.d\ prior = \sqrt{\frac{(y^0)^2}{1-\frac{1}{n^0}}} \quad (13)$$

$$s.d\ posterior = \sqrt{\frac{(y^n)^2}{1-\frac{1}{n^n}}} \quad (14)$$

Address the problem of prior data conflict

Although this is a problem of prior-data conflict, a model can be presented to address the problem of prior-data conflict , through the submission of a set of parameters prior as suggested by [5]. Through the following model $\prod^0 = n^0 x[\underline{y}^0, \overline{y}^0]$. Another suggestion was made by [7] **.** For result a set of prior parameters through the model $\prod^0 = [\underline{n}^0, \overline{n}^0] x[\underline{y}^0, \overline{y}^0]$. In general the model submitted for resulting a set of prior parameters is called **(generalized iLuck-Model).** Then we get the updated parameters to be more accurate as shown in the following steps [10] **:**

$$\boldsymbol{f_1\left(\lambda/\underline{n}^0, \underline{y}^0\right) = \frac{(\underline{n}^0 \underline{y}^0)^{\underline{n}^0+1}}{\Gamma(\underline{n}^0+1)} \lambda^{-(\underline{n}^0+1)-1} e^{-\frac{\underline{n}^0 \underline{y}^0}{\lambda}}} \quad (15)$$

$$\boldsymbol{f_2\left(\lambda/\underline{n}^0, \overline{y}^0\right) = \frac{(\underline{n}^0 \overline{y}^0)^{\underline{n}^0+1}}{\Gamma(\underline{n}^0+1)} \lambda^{-(\underline{n}^0+1)-1} e^{-\frac{\underline{n}^0 \overline{y}^0}{\lambda}}} \quad (16)$$

$$\boldsymbol{f_3\left(\lambda/\overline{n}^0, \underline{y}^0\right) = \frac{(\overline{n}^0 \underline{y}^0)^{\overline{n}^0+1}}{\Gamma(\overline{n}^0+1)} \lambda^{-\left(\overline{n}^0+1\right)-1} e^{-\frac{\overline{n}^0 \underline{y}^0}{\lambda}}} \quad (17)$$

$$\boldsymbol{f_4\left(\lambda/, \overline{n}^0, \overline{y}^0\right) = \frac{(\overline{n}^0 \overline{y}^0)^{\overline{n}^0+1}}{\Gamma(\overline{n}^0+1)} \lambda^{-\left(\overline{n}^0+1\right)-1} e^{-\frac{\overline{n}^0 \overline{y}^0}{\lambda}}} \quad (18)$$

$\boldsymbol{\underline{n}^0}$:lower ، $\boldsymbol{\overline{n}^0}$:upper

$\underline{y}^{0}$:lower ‘$\overline{y}^{0}$:upper

The equations (15)-(18) represent a set of prior distributions & a set of posterior distributions is obtained as shown below:

$$f_1(\lambda\backslash t)=\frac{(\underline{n}^0\underline{y}^0+\tau(t))^{\underline{n}^0+n+1}}{\Gamma(\underline{n}^0+n+1)}\lambda^{-(\underline{n}^0+n+1)-1}e^{-\frac{(\underline{n}^0\underline{y}^0+\tau(t))}{\lambda}} \quad (19)$$

$$f_2(\lambda\backslash t)=\frac{(\underline{n}^0\overline{y}^0+\tau(t))^{\underline{n}^0+n+1}}{\Gamma(\underline{n}^0+n+1)}\lambda^{-(\underline{n}^0+n+1)-1}e^{-\frac{(\underline{n}^0\overline{y}^0+\tau(t))}{\lambda}} \quad (20)$$

$$f_3(\lambda\backslash t)=\frac{(\overline{n}^0\underline{y}^0+\tau(t))^{\overline{n}^0+n+1}}{\Gamma(\overline{n}^0+n+1)}\lambda^{-(\overline{n}^0+n+1)-1}e^{-\frac{(\overline{n}^0\underline{y}^0+\tau(t))}{\lambda}} \quad (21)$$

$$f_4(\lambda\backslash t)=\frac{(\overline{n}^0\overline{y}^0+\tau(t))^{\overline{n}^0+n+1}}{\Gamma(\overline{n}^0+n+1)}\lambda^{-(\overline{n}^0+n+1)-1}e^{-\frac{(\overline{n}^0\overline{y}^0+\tau(t))}{\lambda}} \quad (22)$$

The equations (19) -(22) represent a set of posterior distributions & after taking the averages for those posterior distributions we get the iLuck-Model as shown below:

$$\underline{y}^n = lower(y^n) = \begin{cases} \dfrac{\overline{n}^0\underline{y}^0+\tau(x)}{\overline{n}^0+n} & if \quad \overline{\tau}(x)\geq \underline{y}^0 \\ \dfrac{\underline{n}^0\underline{y}^0+\tau(x)}{\underline{n}^0+n} & if \quad \overline{\tau}(x)< \underline{y}^0 \end{cases}$$

$$\overline{y}^n = upper(y^n) = \begin{cases} \dfrac{\overline{n}^0\overline{y}^0+\tau(x)}{\overline{n}^0+n} & if \quad \overline{\tau}(x)\leq \overline{y}^0 \\ \dfrac{\underline{n}^0\overline{y}^0+\tau(x)}{\underline{n}^0+n} & if \quad \overline{\tau}(x)> \overline{y}^0 \end{cases}$$

The posterior distribution will be final & as follows: (what does that mean)

$$f(\lambda/n^m y^m)=\frac{(n^m y^m)^{n^m+1}}{\Gamma(n^m+1)}\lambda^{-(n^m+1)-1}e^{-\frac{n^m y^m}{\lambda}} \quad (23)$$

$$n^m=\frac{lower(n^n)+upper(n^n)}{2} \quad , \quad y^m=\frac{lower(y^n)+upper(y^n)}{2}$$

Robust Bayesian To Estimate The Scale Parameter λ

From equation (23), the Bayesian estimator can be obtained for scale parameter λ under quadratic loss function which is the mean of the posterior distribution as shown below:

$$\hat{\lambda}_{Rob}=\frac{n^m y^m}{n^m} \quad (24)$$

Robust Bayesian To Estimate The Survival Function

From equation (23), the Bayesian estimator can be obtained for survival function under quadratic loss function which is the mean of the posterior distribution as shown below:

$$\hat{S}_{Rob}(t) = \left(\frac{n^m y^m}{n^m y^m + t^\beta}\right)^{n^m+1} \qquad (25)$$

Steps of the simulation experiment

The program was written using R & according to the following steps [1]:

The first step

This step is one of the basic steps in which the default values are chosen as in the following steps: Different default values were chosen for scale parameter λ and shape parameter β. The prior distribution parameters (a, b).

1. Three different samples were selected as follows:
   n: 10,20,40
2. The frequency of the experiment was equal to (1000).

The second step

At this step, data is generated according to the following steps:

Generate the random variable U_i that follows the uniform distribution:

$U = R\&$

Where:

$U_i \sim U(0,1) \quad , \qquad i = 1,2,\ldots,n$

Random variable U is a random variable that describes a model under study using a statistical mathematical method. This method is used to generate various random variables that follows the various probability distributions. This method is characterized by its ease & efficiency:

$u = F(t)$

$t = F^{-1}(u)$

The random variable that follows the Weibull distribution is generated based on the above steps & is as follows:

$\boldsymbol{t = e^{[\log(-\lambda * \log(1-u))]/\beta}}$

Third Step

The survival function and the scale parameter are estimated according to the Bayesian method & the robust Bayesian method.

The fourth step

The estimation methods are compared by using the following measures:

$$\mathrm{IMSE}(\hat{\lambda}) = \frac{1}{r}\sum_{i=1}^{r}(\hat{\lambda}-\lambda)^2$$

$$\mathrm{IMSE}[\hat{S}(t)] = \frac{1}{r}\sum_{i=1}^{r}\left\{\frac{1}{n_t}\sum_{j=1}^{n_t}\left[\hat{S}_i(t_j) - S(t_j)\right]^2\right\}$$

Where:

r: Represents the frequency of experiment.

n_t : Represents the sample size for each experiment (t_i)

The simulation results will then be analyzed to estimate the scale parameter & the survival function of the Weibull distribution & according to the following tables as follows:

Table (1) Integrated mean square error (IMSE) for the scale parameter λ under prior data unconflict

β=2 Lower 2				**n^0 upper 4**				
y^0		λ	n	Regular Bayes	Robust Bayes	s.d prior	s.d posterior	best
Lower	Upper							
1.5	2.5	1.5	10	0.182031	0.434197	3.181981	4.124483	Regular Bayes
			20	0.169029	0.302033	3.181981	3.895044	
			40	0.141357	0.204936	3.181981	3.642233	
2	3	2	10	0.345058	0.739248	5.656854	7.440964	
			20	0.321076	0.524764	5.656854	6.973861	
			40	0.259569	0.356398	5.656854	6.508075	
2.5	3.5	2.5	10	0.559322	1.129046	8.838835	11.69037	
			20	0.488893	0.774301	8.838835	10.85684	
			40	0.389744	0.523473	8.838835	10.10827	
β=3 Lower 3				n^0 upper 5				
y^0		λ	n	Regular Bayes	Robust Bayes	s.d prior	s.d posterior	best
Lower	Upper							

Table (1) Integrated mean square error (IMSE) for the scale parameter λ under prior data unconflict

1.5	2.5	1.5	10	0.119066	0.338307	2.755676	3.695658	Regular Bayes
			20	0.104449	0.218675	2.755676	3.476381	
			40	0.078498	0.131644	2.755676	3.248046	
2	3	2	10	0.225798	0.552795	4.898979	6.625628	
			20	0.181882	0.346918	4.898979	6.157614	
			40	0.141076	0.218842	4.898979	5.771726	
2.5	3.5	2.5	10	0.333132	0.770847	7.654655	10.23463	
			20	0.277153	0.503538	7.654655	9.619343	
			40	0.217270	0.324069	7.654655	9.017049	
β=4				**n^0**				
Lower				**upper**				
4				**6**				
y^0		**λ**	**n**	**Regular Bayes**	**Robust Bayes**	**s.d prior**	**s.d posterior**	**best**
Lower	**Upper**							
1.5	2.5	1.5	10	0.093425	0.300260	2.598076	3.468844	Regular Bayes
			20	0.070829	0.176836	2.598076	3.249739	
			40	0.063769	0.116070	2.598076	3.117757	
2	3	2	10	0.158241	0.449122	4.618802	6.15736	
			20	0.134422	0.288341	4.618802	5.822997	
			40	0.109040	0.183461	4.618802	5.524409	
2.5	3.5	2.5	10	0.263884	0.666657	7.216878	9.713252	
			20	0.233611	0.446754	7.216878	9.182148	
			40	0.176273	0.277789	7.216878	8.654652	

Table (2) Integrated mean square error (IMSE) for the survival function under prior data unconflict

β=2			**n^0**			
Lower			**upper**			
2			**4**			
y^0		**λ**	**n**	**Bayes Survival**	**Robust Survival**	**best**
Lower	Upper					
1.5	2.5	1.5	10	0.005336	0.007356	Bayes Survival
			20	0.004834	0.006014	
			40	0.003925	0.004560	
2	3	2	10	0.005701	0.007092	
			20	0.004973	0.005770	
			40	0.004037	0.004470	
2.5	3.5	2.5	10	0.005853	0.006928	
			20	0.004949	0.005541	
			40	0.003925	0.004239	

***Cont...* Table (2) Integrated mean square error (IMSE) for the survival function under prior data unconflict**

β=3 Lower 3			**n^0**			
			upper			
			5			
y^0		**λ**	**n**	**Bayes Survival**	**Robust Survival**	**best**
Lower	**Upper**					
1.5	2.5	1.5	10	0.003737	0.006041	Bayes Survival
			20	0.003113	0.004432	
			40	0.002326	0.003017	
2	3	2	10	0.003811	0.005392	
			20	0.003027	0.003933	
			40	0.002350	0.002832	
2.5	3.5	2.5	10	0.003697	0.004886	
			20	0.003040	0.003729	
			40	0.002302	0.002663	
β=4 Lower 4			n^0			
			upper			
			6			

Table (3) Integrated mean square error (IMSE) for the scale parameter λ under prior data conflict

β=2 Lower 2				**n^0**				
				upper				
				4				
y^0		λ	n	Regular Bayes	Robust Bayes	s.d prior	s.d posterior	best
Lower	Upper							
1.5	2.5	1.5	10	0.104965	0.053542	3.181981	1.934395	Robust Bayes
			20	0.069947	0.045689	3.181981	2.004401	
			40	0.034577	0.028687	3.181981	2.149770	
2	3	2	10	0.207645	0.103980	5.656854	3.317023	
			20	0.116619	0.081806	5.656854	3.635705	
			40	0.061550	0.053343	5.656854	3.865050	
2.5	3.5	2.5	10	0.312751	0.172314	8.838835	5.298248	
			20	0.175169	0.123418	8.838835	5.677780	
			40	0.096588	0.081201	8.838835	5.959071	
β=3 Lower 3				n^0				
				upper				
				5				

***Cont...* Table (3) Integrated mean square error (IMSE) for the scale parameter λ under prior data conflict**

y^0		λ	n	Regular Bayes	Robust Bayes	s.d prior	s.d posterior	best
Lower	Upper							
1.5	2.5	1.5	10	0.110766	0.038602	2.755676	1.782415	
			20	0.061015	0.030741	2.755676	1.939564	
			40	0.036828	0.024552	2.755676	2.033448	
2	3	2	10	0.185234	0.072868	4.898979	3.234684	
			20	0.122498	0.066652	4.898979	3.381657	Robust Bayes
			40	0.065156	0.044440	4.898979	3.596966	
2.5	3.5	2.5	10	0.286239	0.121281	7.654655	5.065197	
			20	0.175785	0.096885	7.654655	5.312925	
			40	0.097294	0.068774	7.654655	5.651768	
β=4				n^0				
Lower				upper				
4				6				
y^0		λ	n	Regular Bayes	Robust Bayes	s.d prior	s.d posterior	best
Lower	Upper							
1.5	2.5	1.5	10	0.099661	0.030929	2.598076	1.785932	
			20	0.061456	0.024841	2.598076	1.879410	
			40	0.034769	0.019242	2.598076	1.986501	
2	3	2	10	0.175046	0.056963	4.618802	3.174600	
			20	0.112765	0.052003	4.618802	3.337410	Robust Bayes
			40	0.071264	0.042612	4.618802	3.462550	
2.5	3.5	2.5	10	0.273468	0.092465	7.216878	4.909911	
			20	0.170271	0.084273	7.216878	5.242145	
			40	0.097597	0.060702	7.216878	5.494434	

Table (4) Integrated mean square error (IMSE) for the survival function under prior data conflict

β=2			**n^0**			
Lower			**upper**			
2			**4**			
y^0		**λ**	**n**	**Bayes Survival**	**Robust Survival**	**best**
Lower	**Upper**					
1.5	2.5	1.5	10	0.004146	0.002141	
			20	0.002745	0.001824	
			40	0.001291	0.001044	
2	3	2	10	0.004687	0.002767	
			20	0.002631	0.001943	Robust Survival
			40	0.001296	0.001110	
2.5	3.5	2.5	10	0.004602	0.003058	
			20	0.002501	0.001937	
			40	0.001304	0.001131	
β=3			n^0			
Lower			upper			
3			5			

***Cont...* Table (4) Integrated mean square error (IMSE) for the survival function under prior data conflict**

y^0		λ	n	Bayes Survival	Robust Survival	best
Lower	Upper					
1.5	2.5	1.5	10	0.004265	0.001691	Robust Survival
			20	0.002293	0.001265	
			40	0.001396	0.000985	
2	3	2	10	0.003937	0.001944	
			20	0.002677	0.001708	
			40	0.001372	0.001035	
2.5	3.5	2.5	10	0.003918	0.002221	
			20	0.002388	0.001627	
			40	0.001304	0.001037	
β=4 Lower 4			n^0			
			upper			
			6			
y^0		λ	n	Bayes Survival	Robust Survival	best
Lower	Upper					
1.5	2.5	1.5	10	0.003674	0.001209	Robust Survival
			20	0.002293	0.001053	
			40	0.001289	0.000790	
2	3	2	10	0.003625	0.001509	
			20	0.002366	0.001321	
			40	0.001493	0.001027	
2.5	3.5	2.5	10	0.003532	0.001697	
			20	0.002315	0.001443	
			40	0.001296	0.000948	

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Conflict of Interest: None to declare.

Ethical Clearance: All experimental protocols were approved under the College of Administration & Economic and all experiments were carried out in accordance with approved guidelines.

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